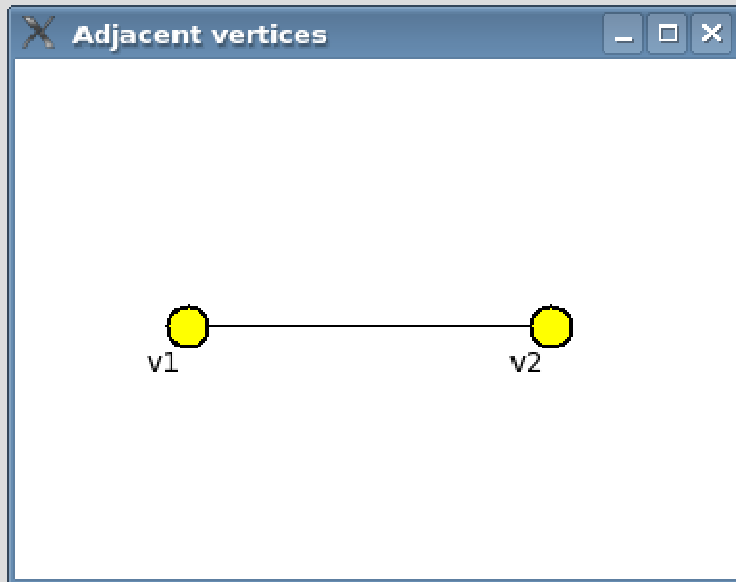


Comparing search algorithms of maximum clique

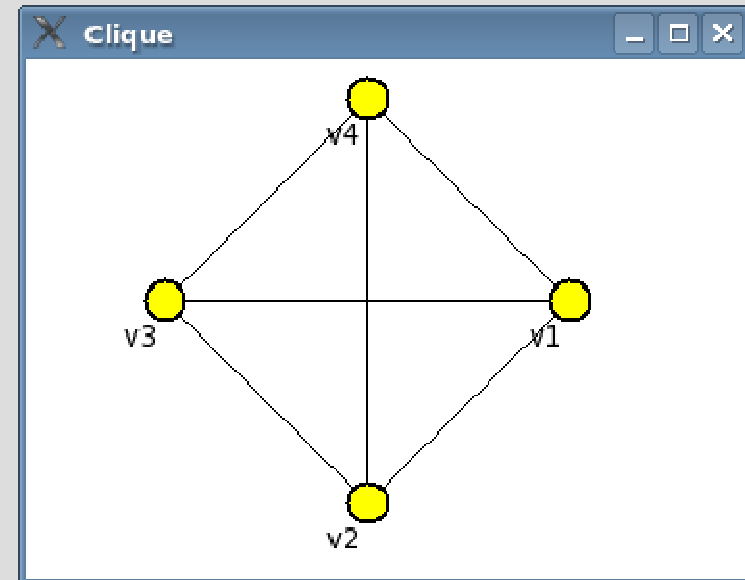
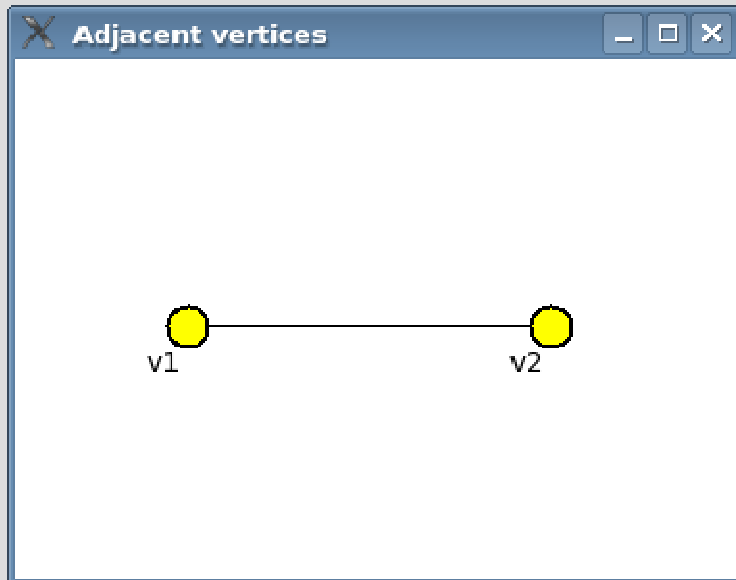
Blagovest Kasabov

Maximum Clique Problem

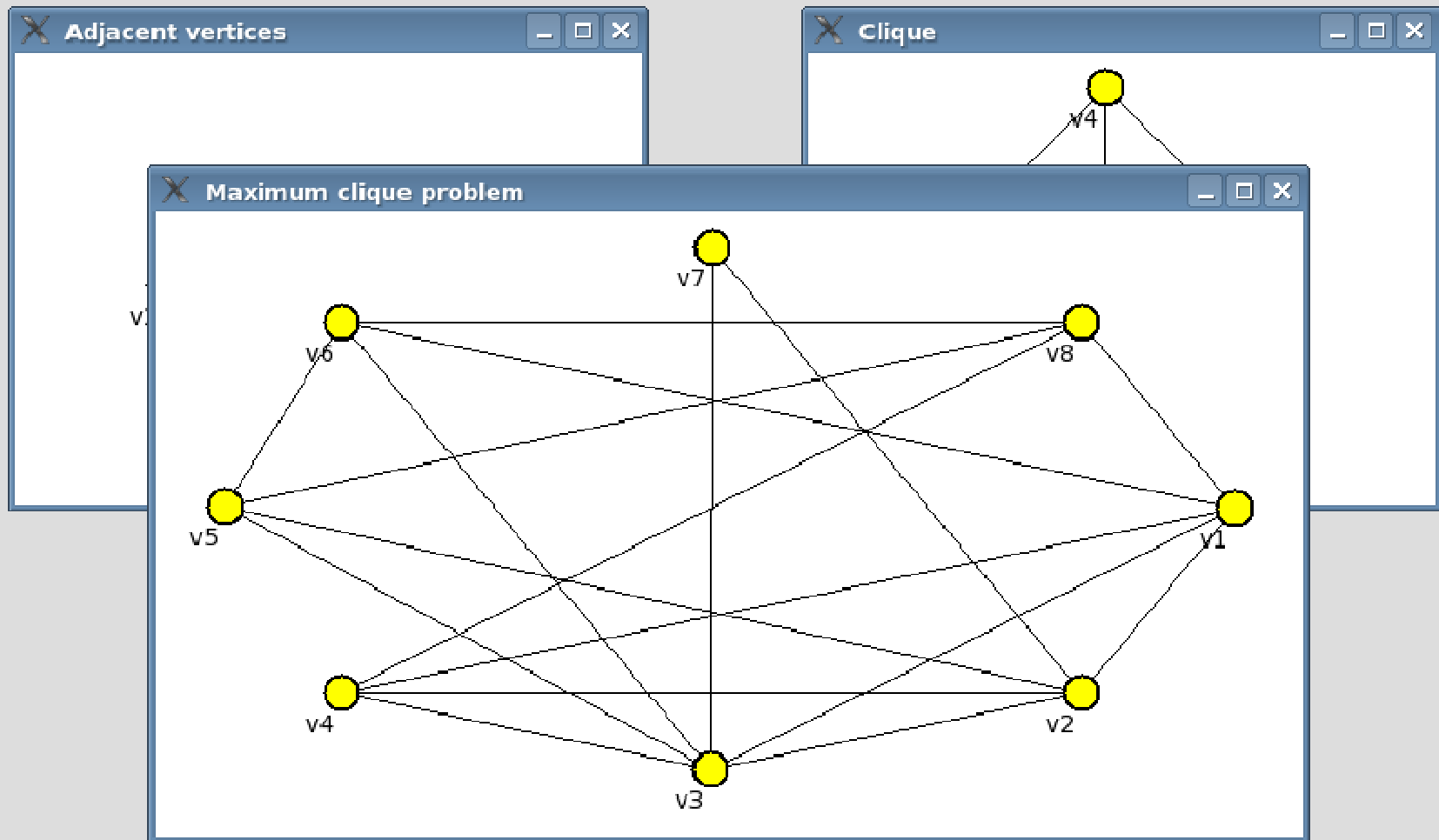
Maximum clique problem



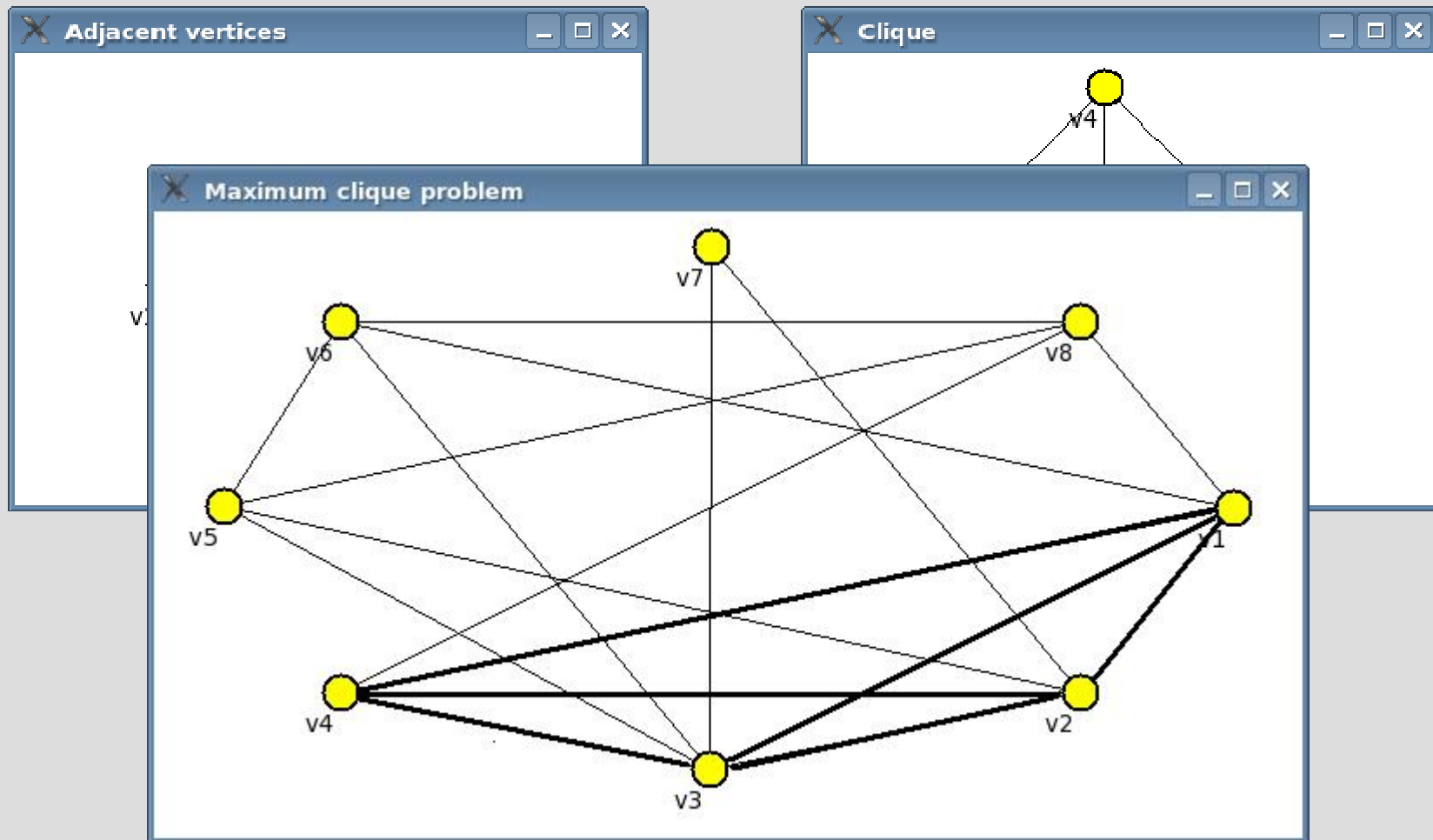
Maximum clique problem



Maximum clique problem



Maximum clique problem



Clique Finders

Undirected graphs

Ostergard's Cliquer

Alignment graphs

Alignment graph Clique Finder (ACF)

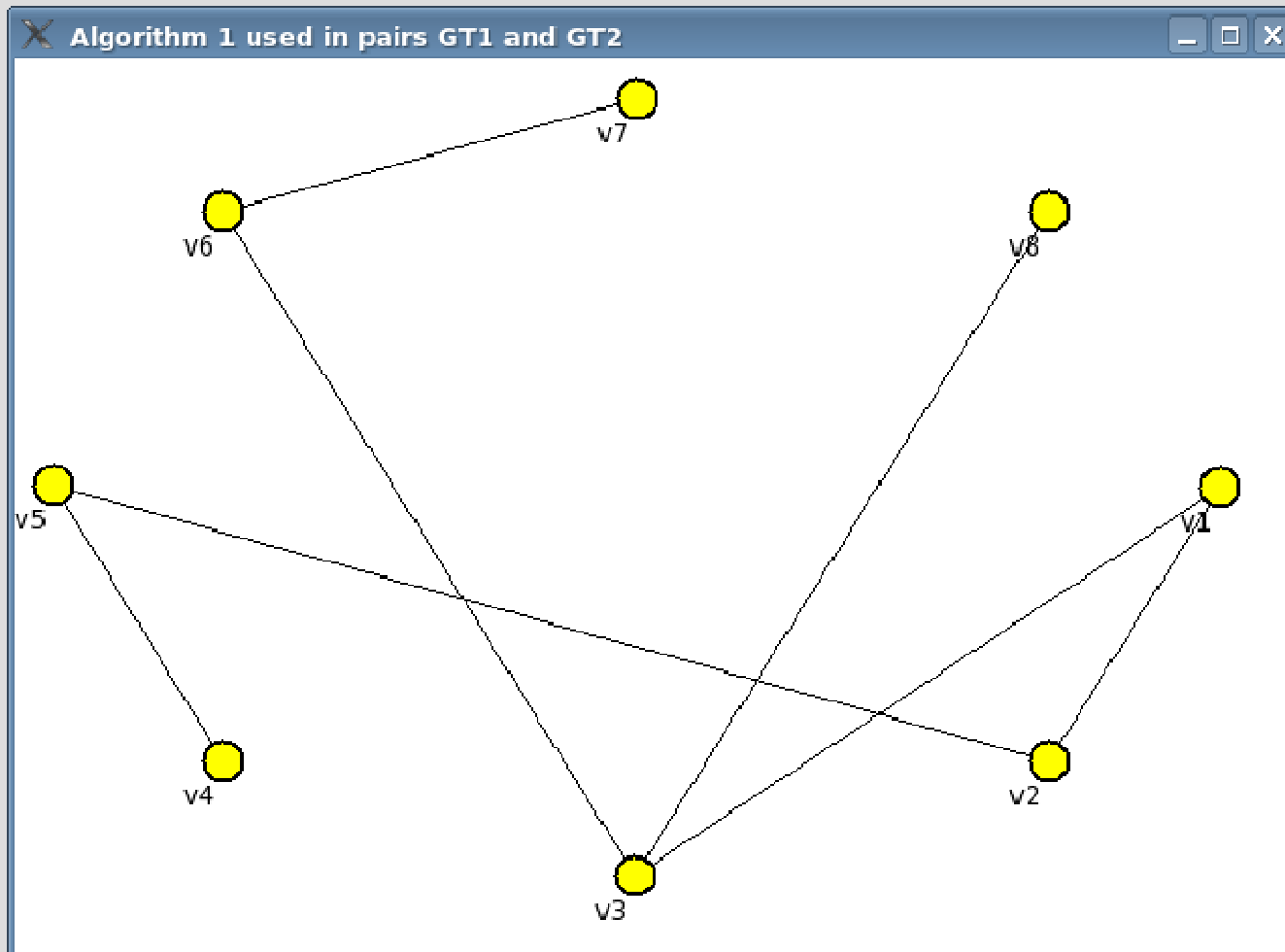
Alignment graphs:

A $m \times n$ alignment graph $G = (V, E)$ is a graph in which the vertex set V is depicted by a m -rows \times n -columns array T , where each cell $T[i][k]$ contains at most one vertex, labeled by $i.k$ from V .

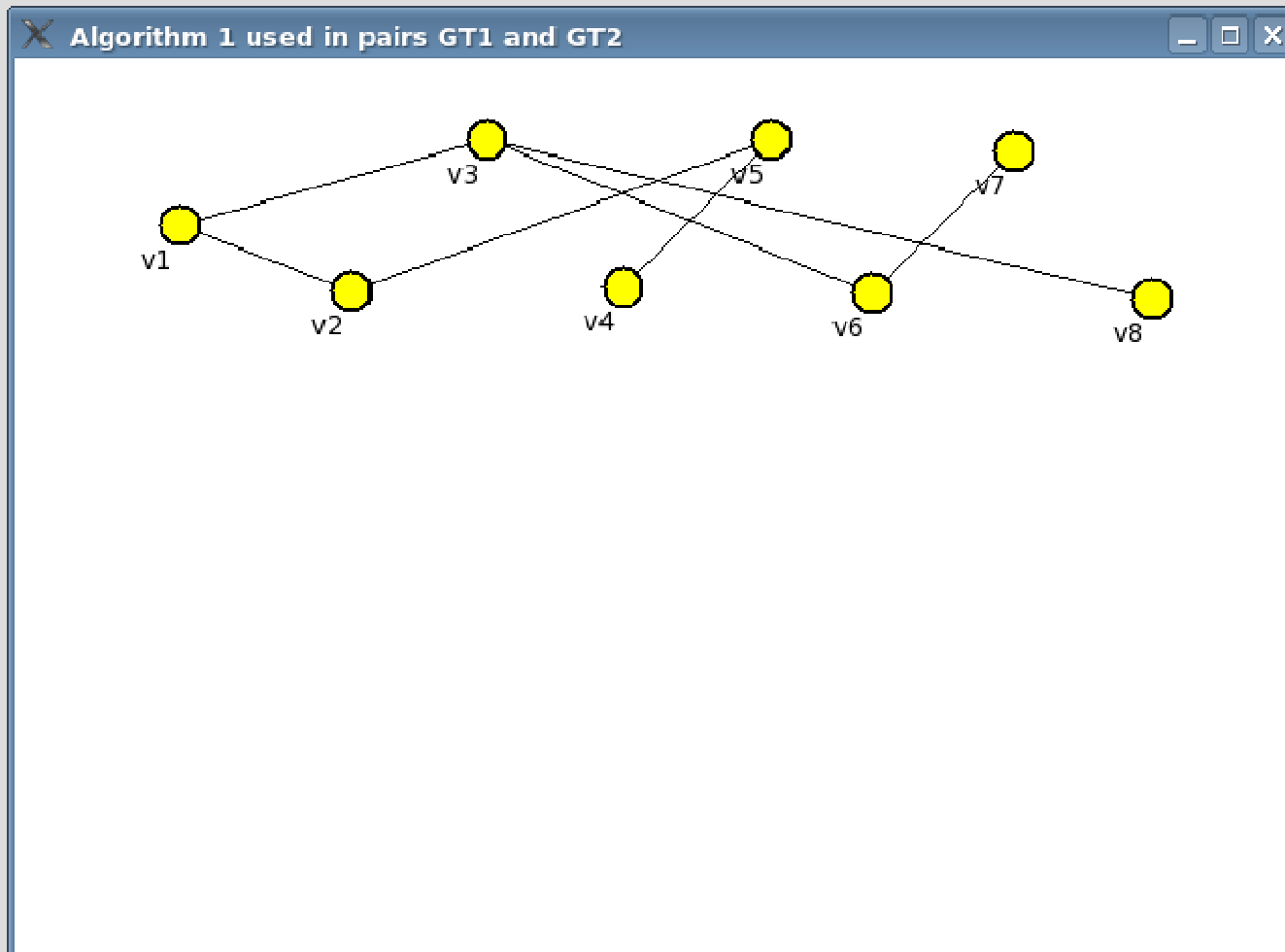
Two vertices $i.k$ and $j.l$ can be connected by an edge $(i.k, j.l)$ in E only if $i < j$ and $k < l$

Algorithms for creating alignment graphs

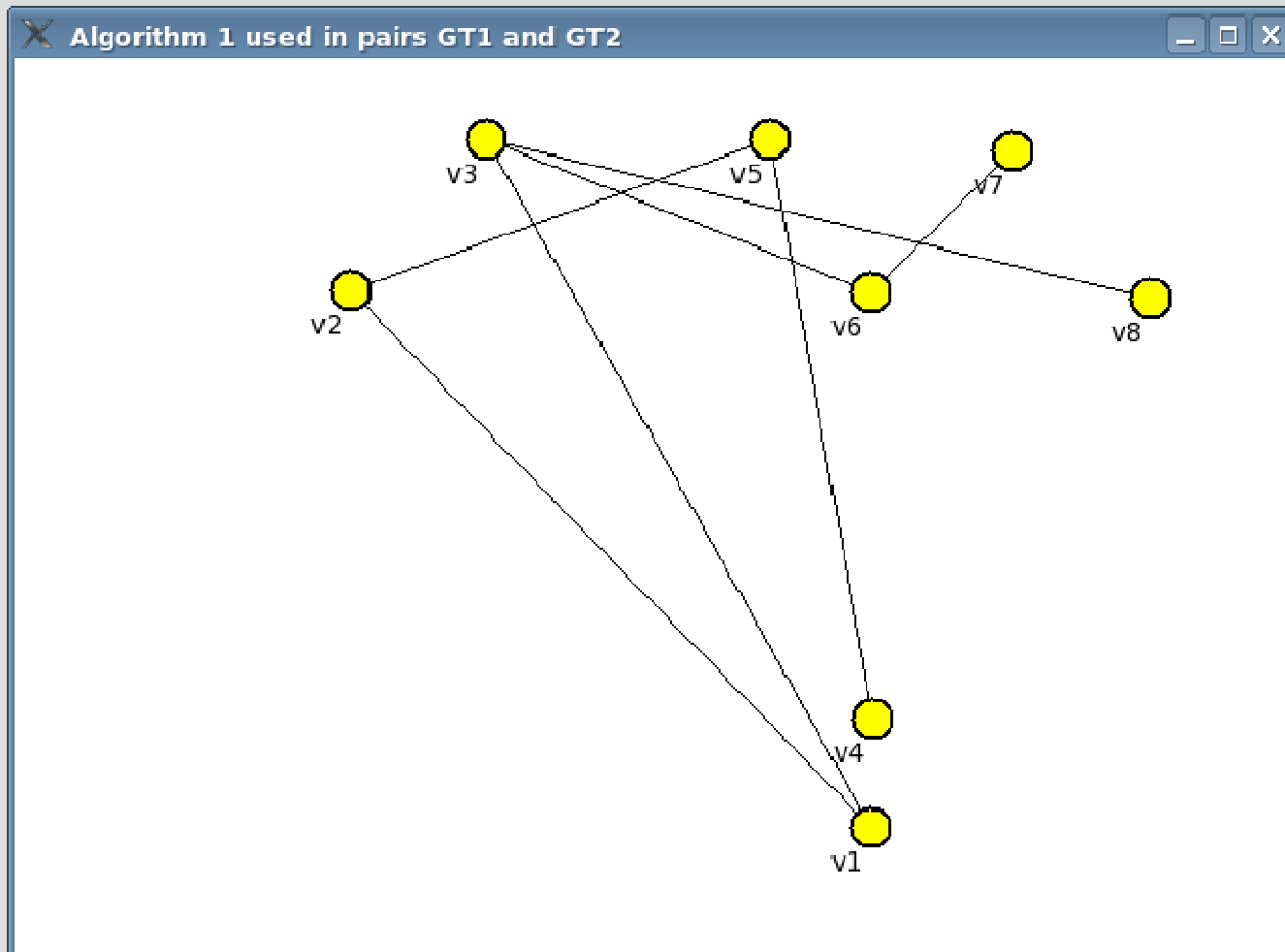
Random Graph



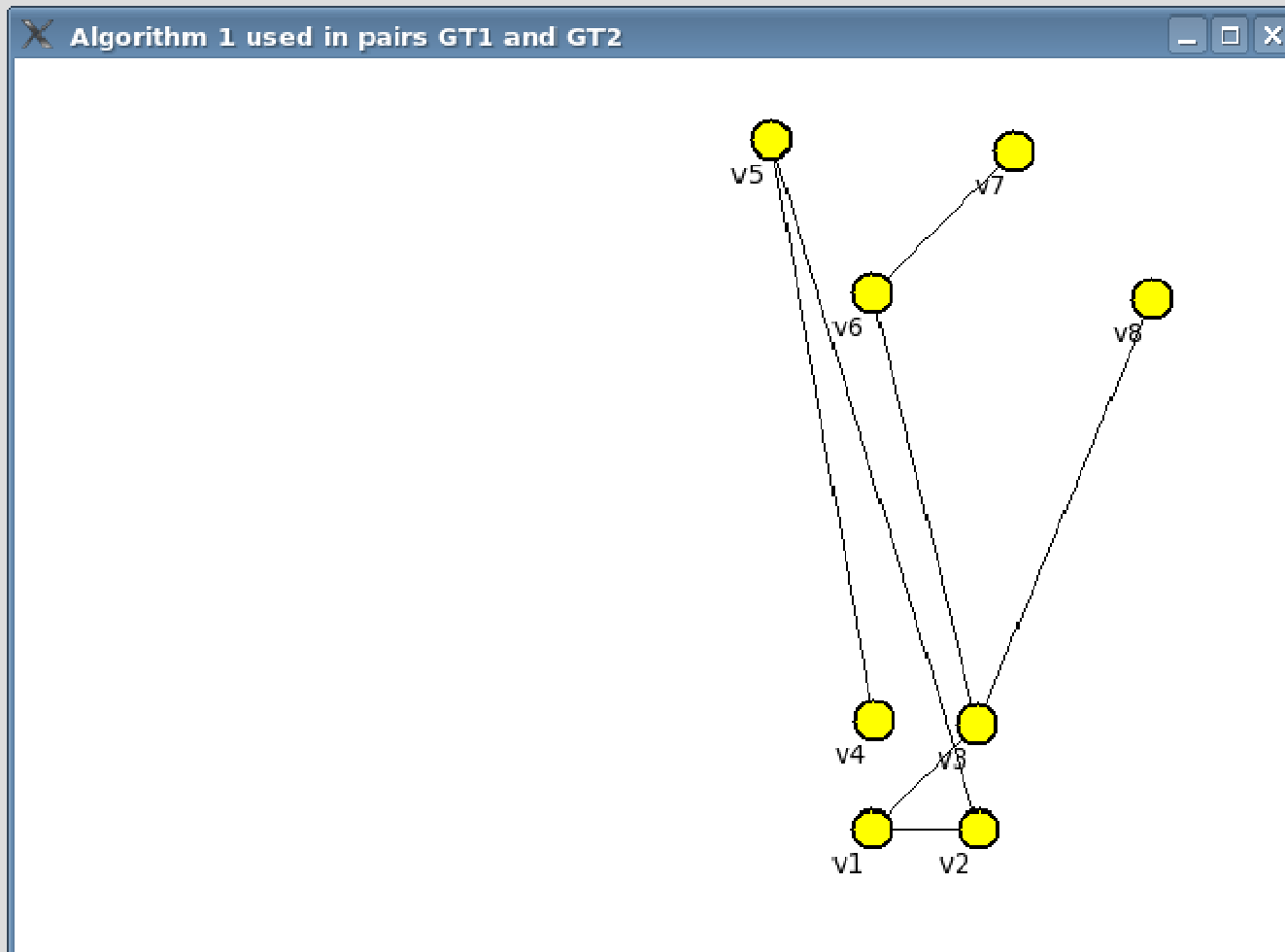
Ordering in Algorithm 1



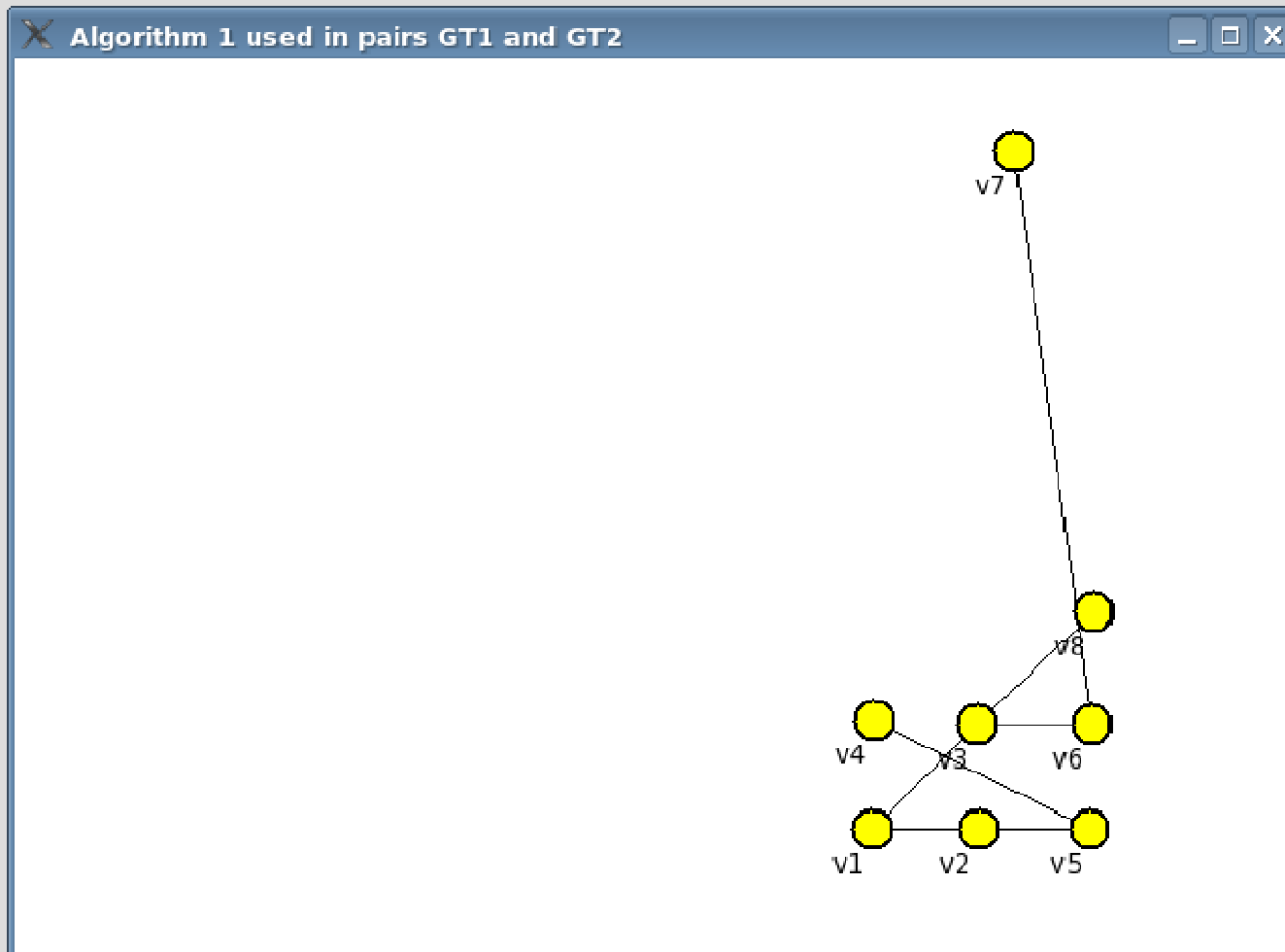
Computing the vertices in 1-st column



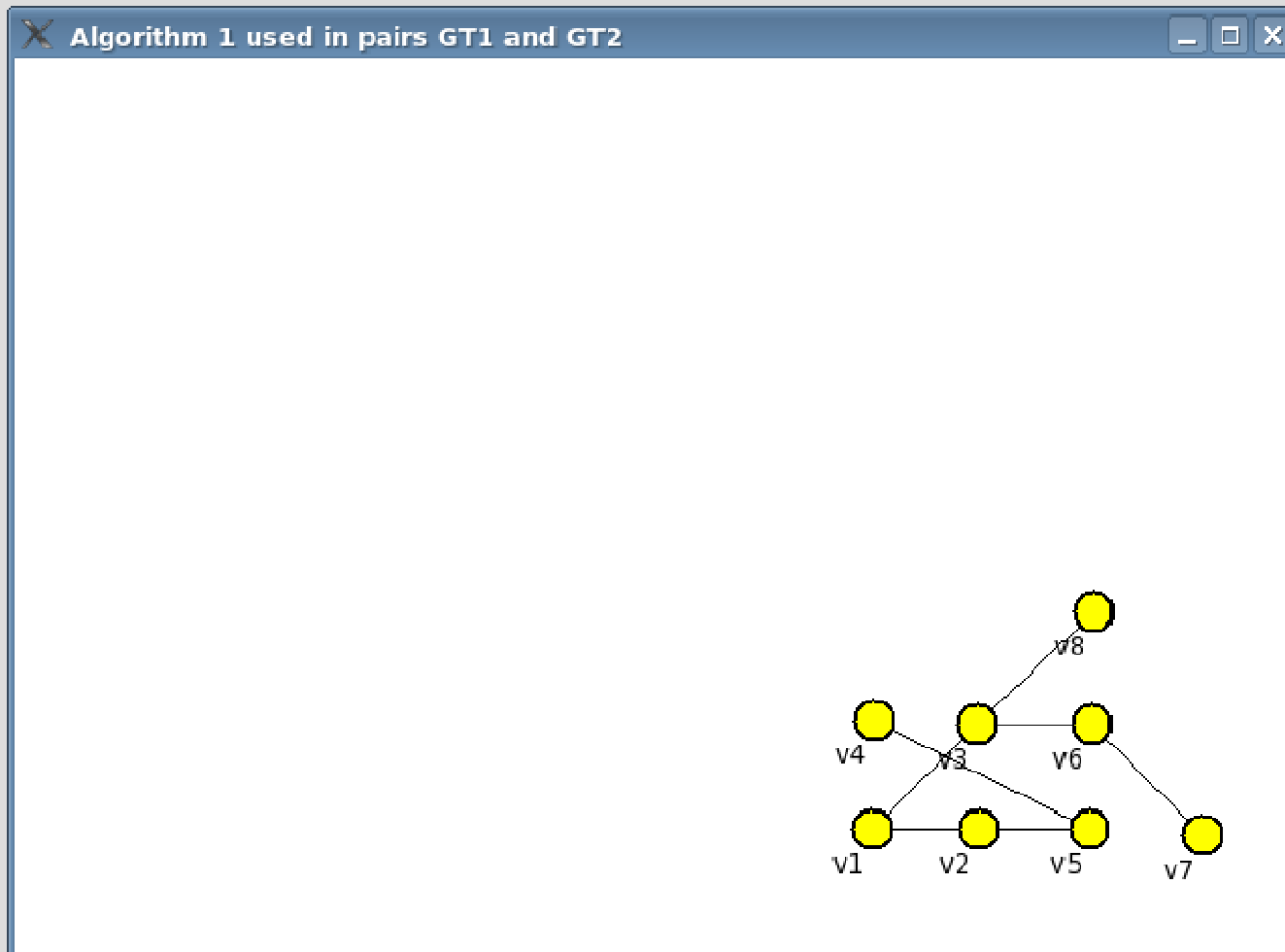
Computing the vertices in 2-nd column



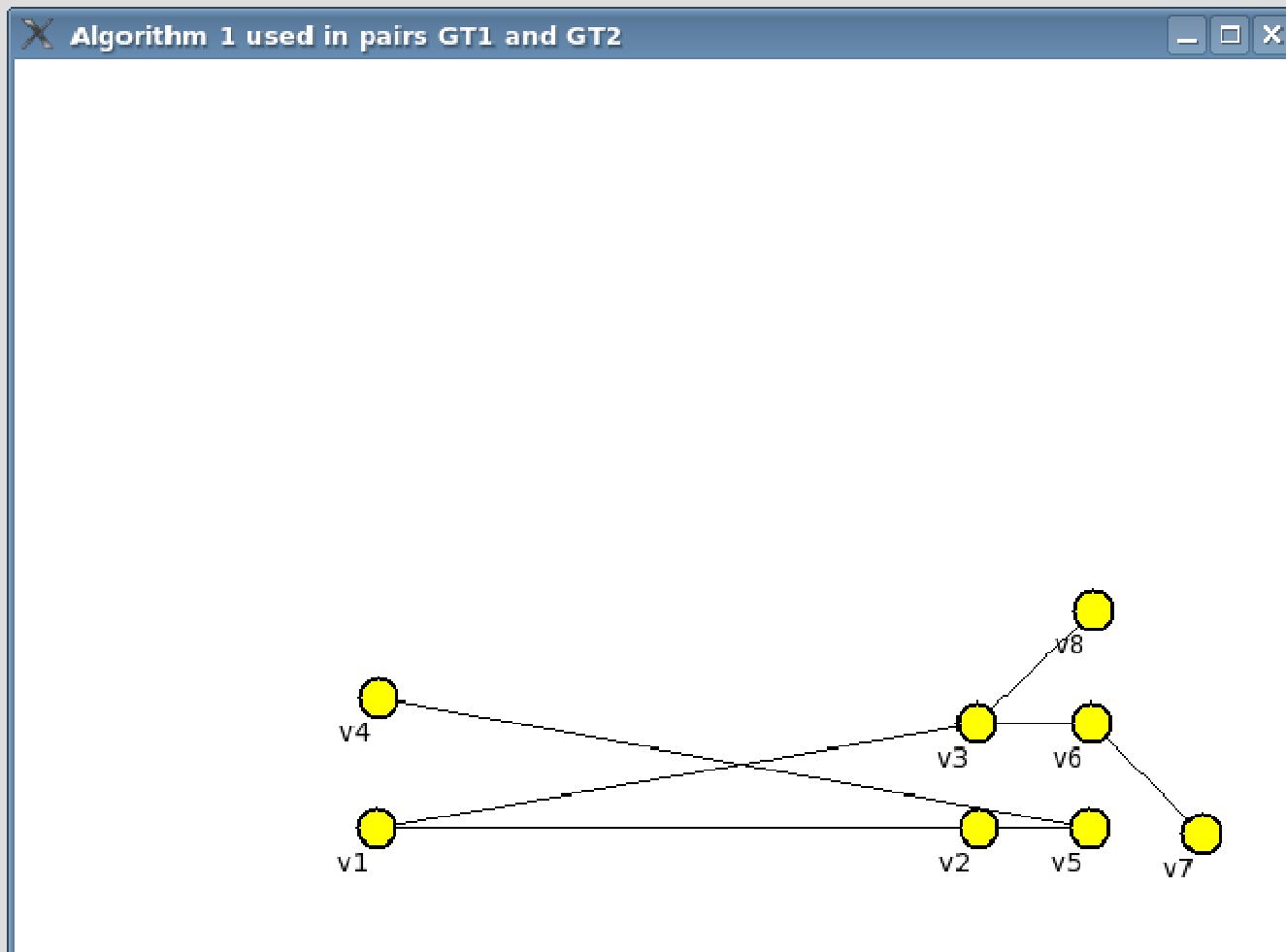
Computing the vertices in 3-rd column



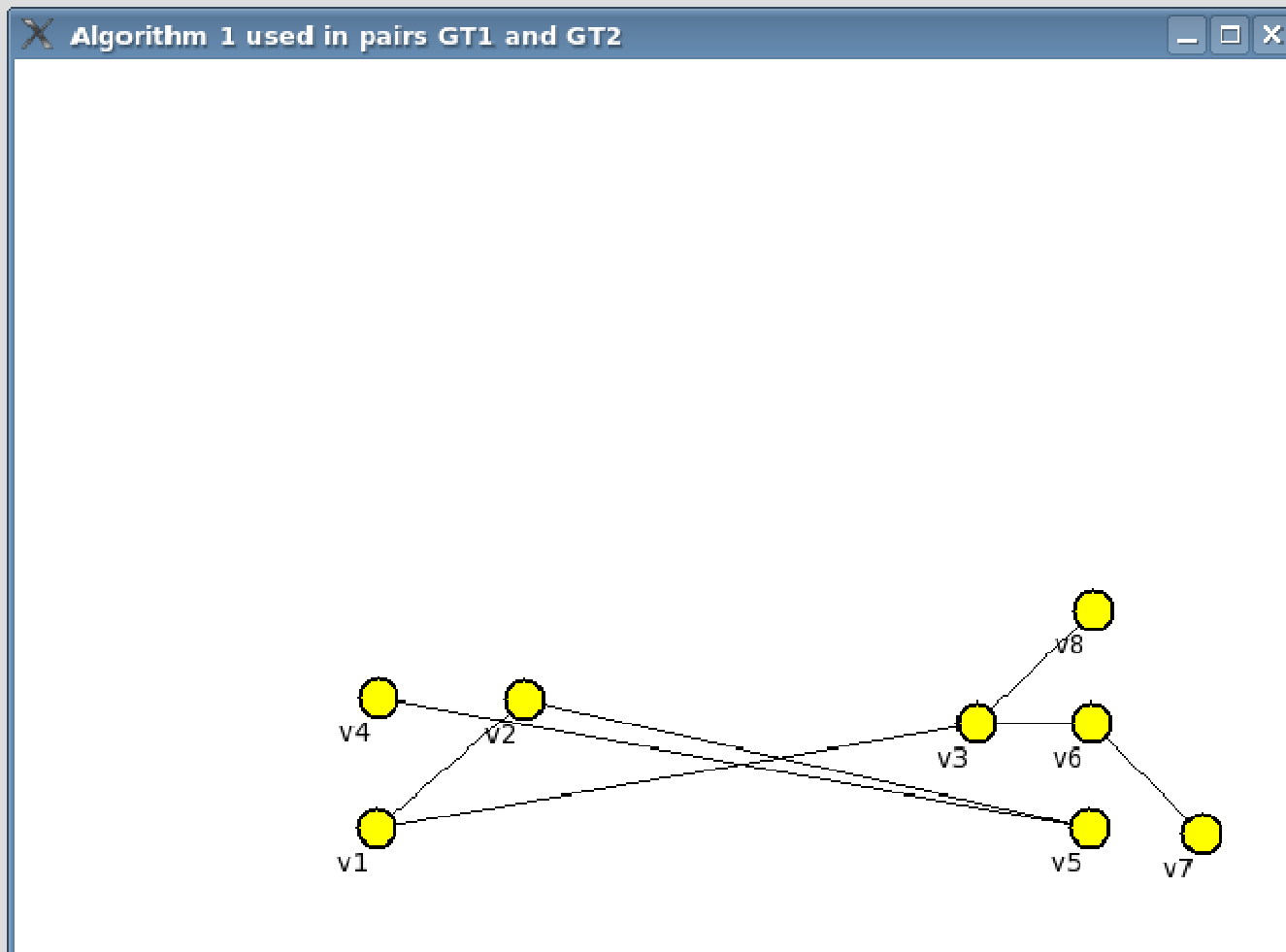
Computing the vertices in the last column



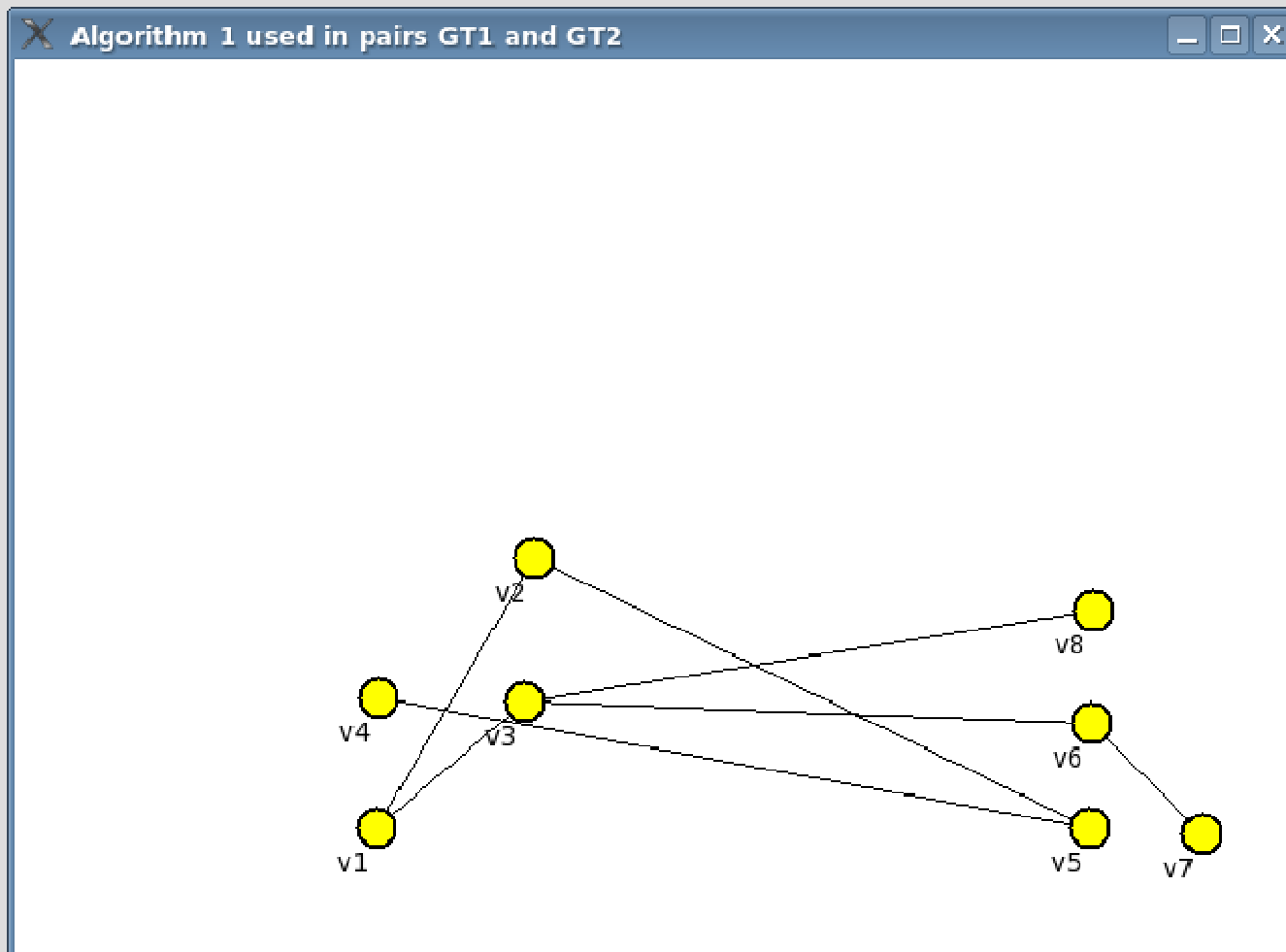
Computing the position of vertices in 1-st column



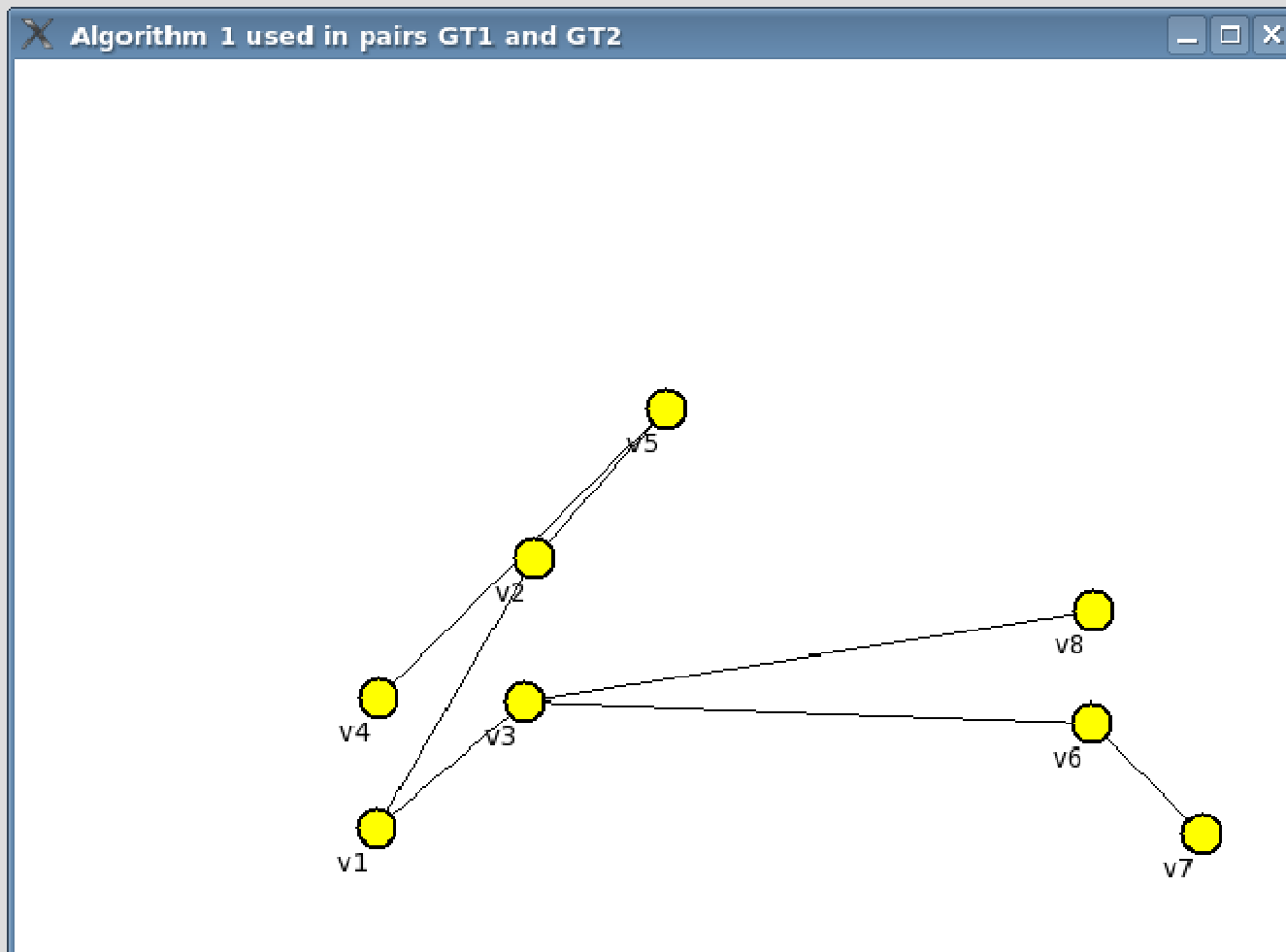
Computing the position of vertices in 2-nd column



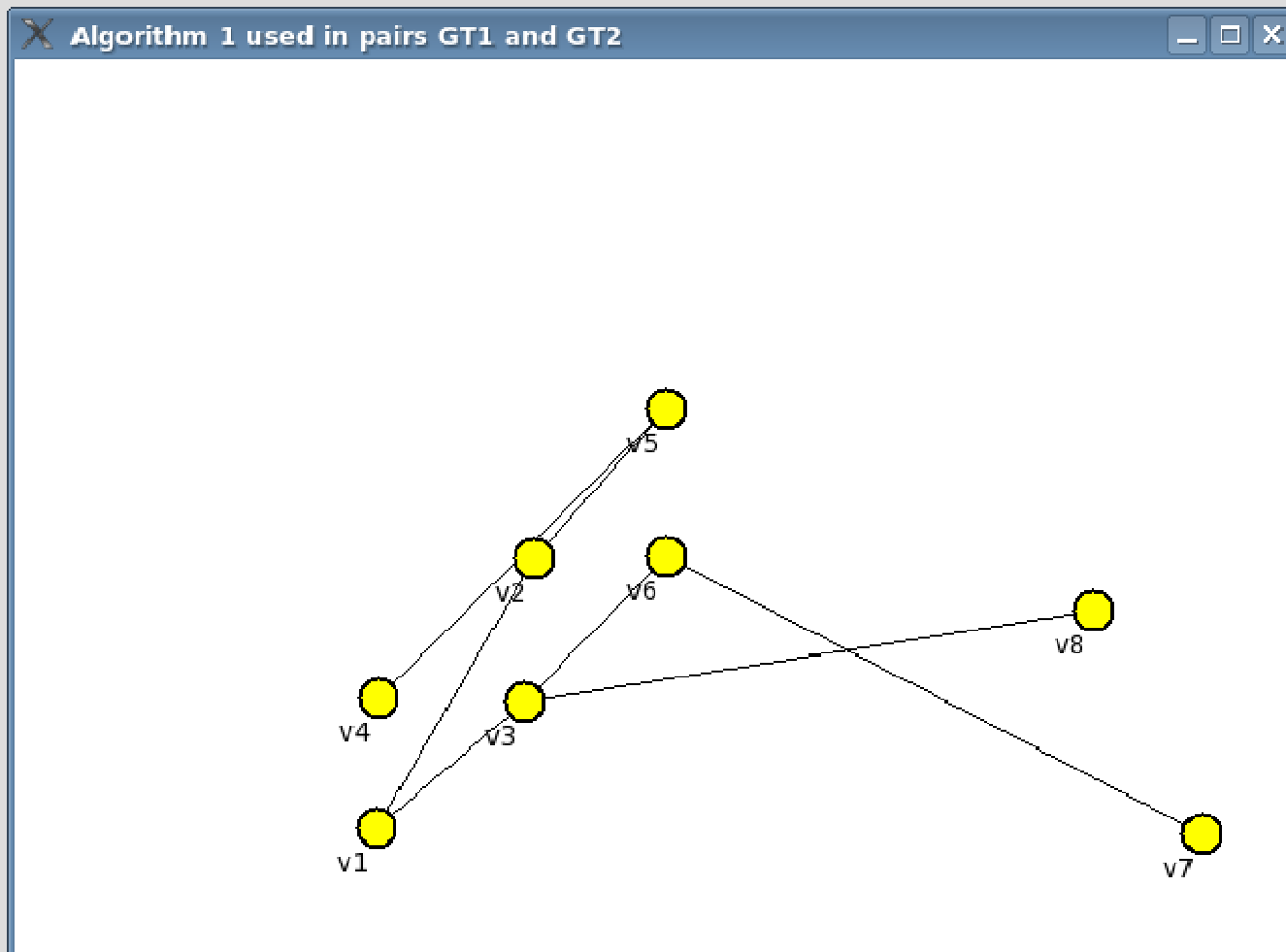
Computing the position of vertices in 2-nd column



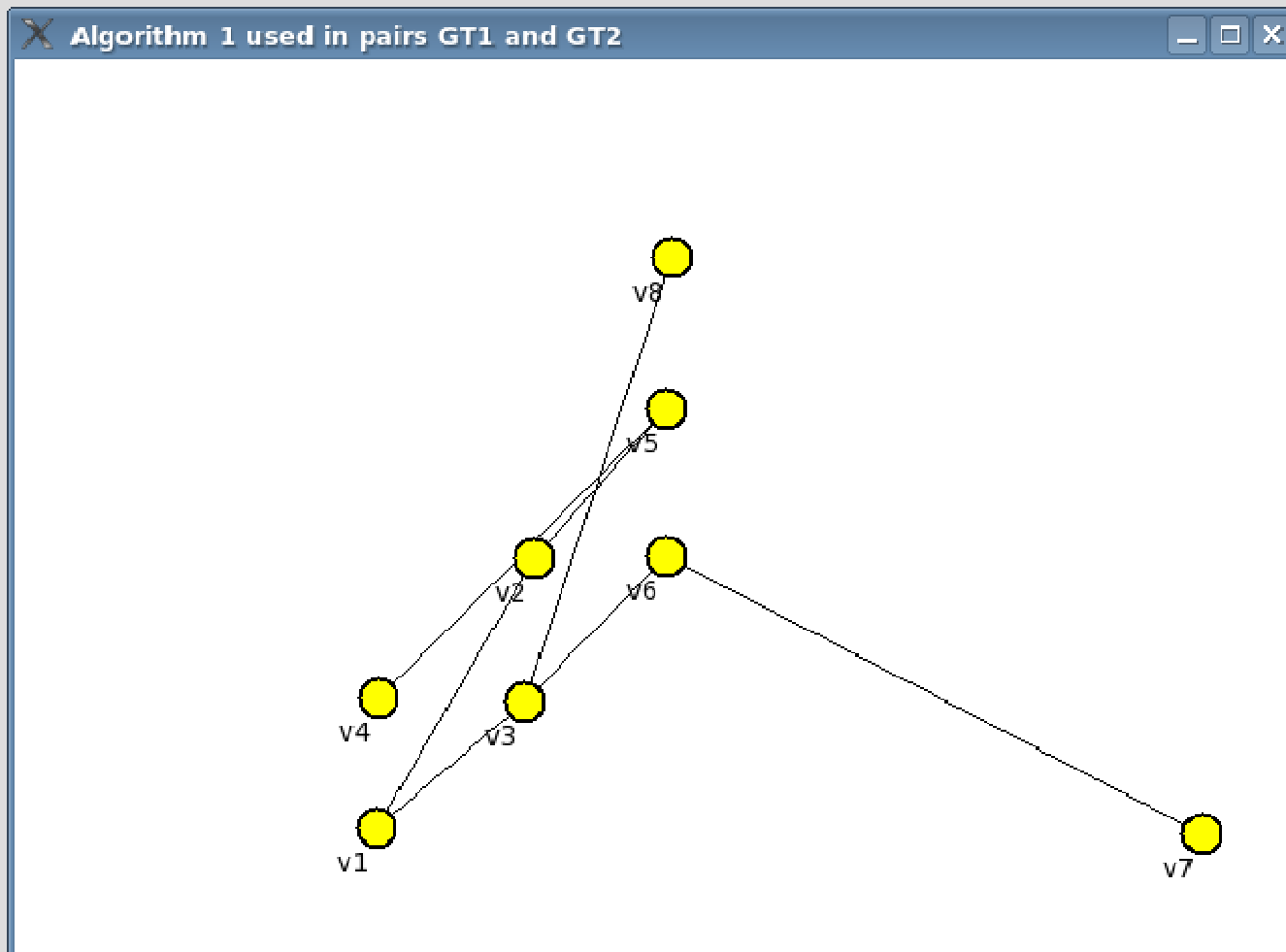
Computing the position of vertices in 3-rd column



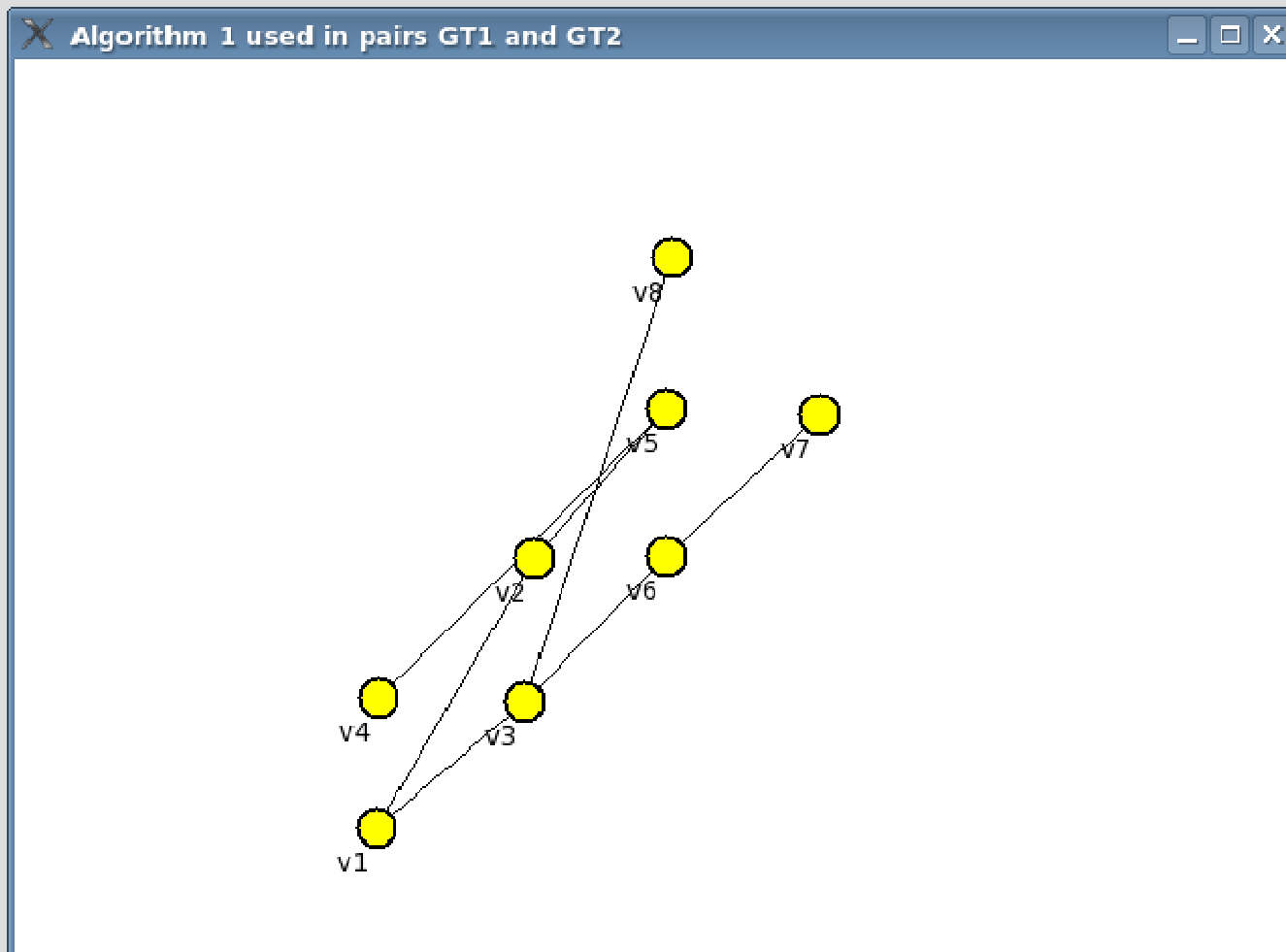
Computing the position of vertices in 3-rd column



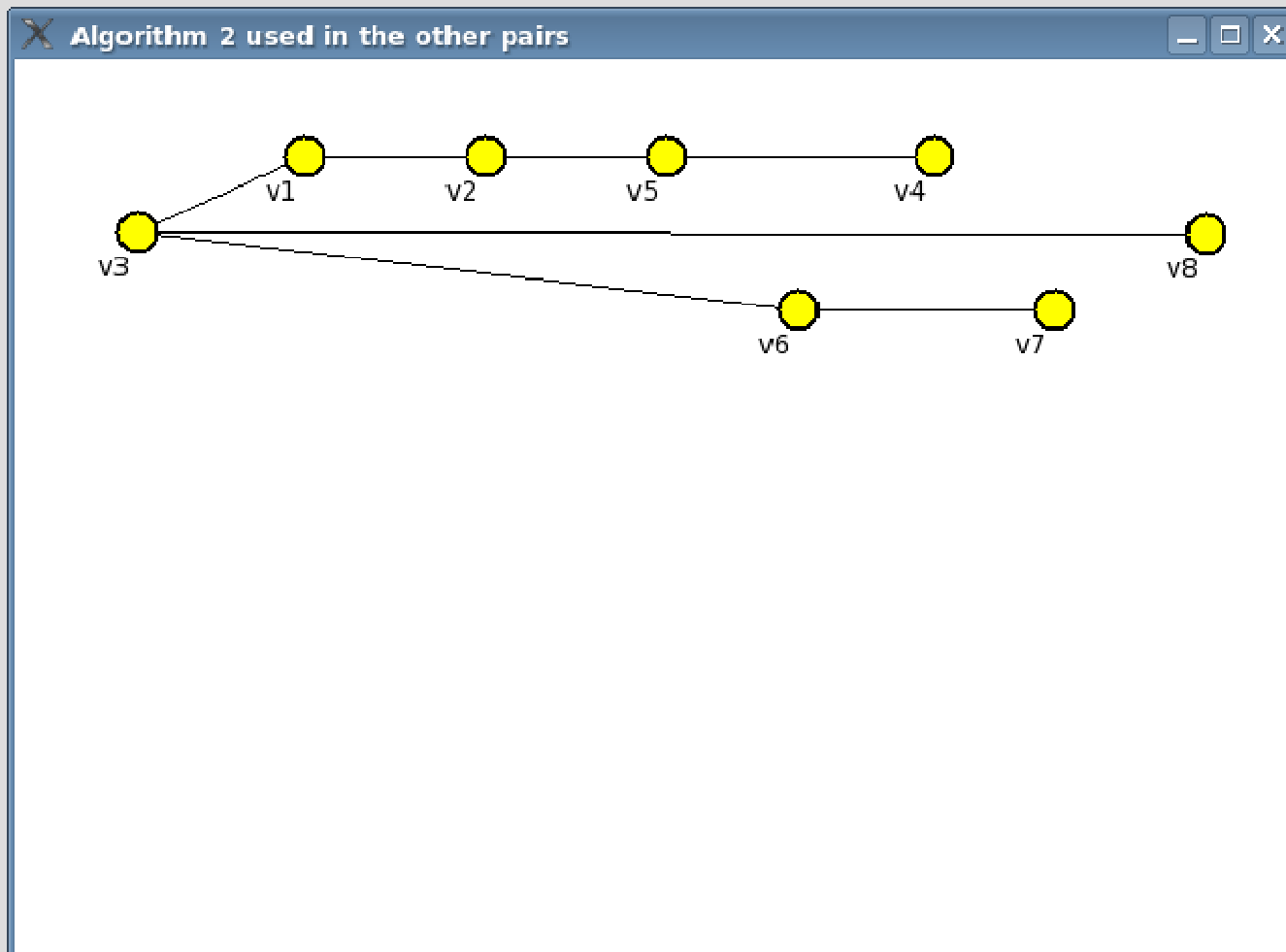
Computing the position of vertices in 3-rd column



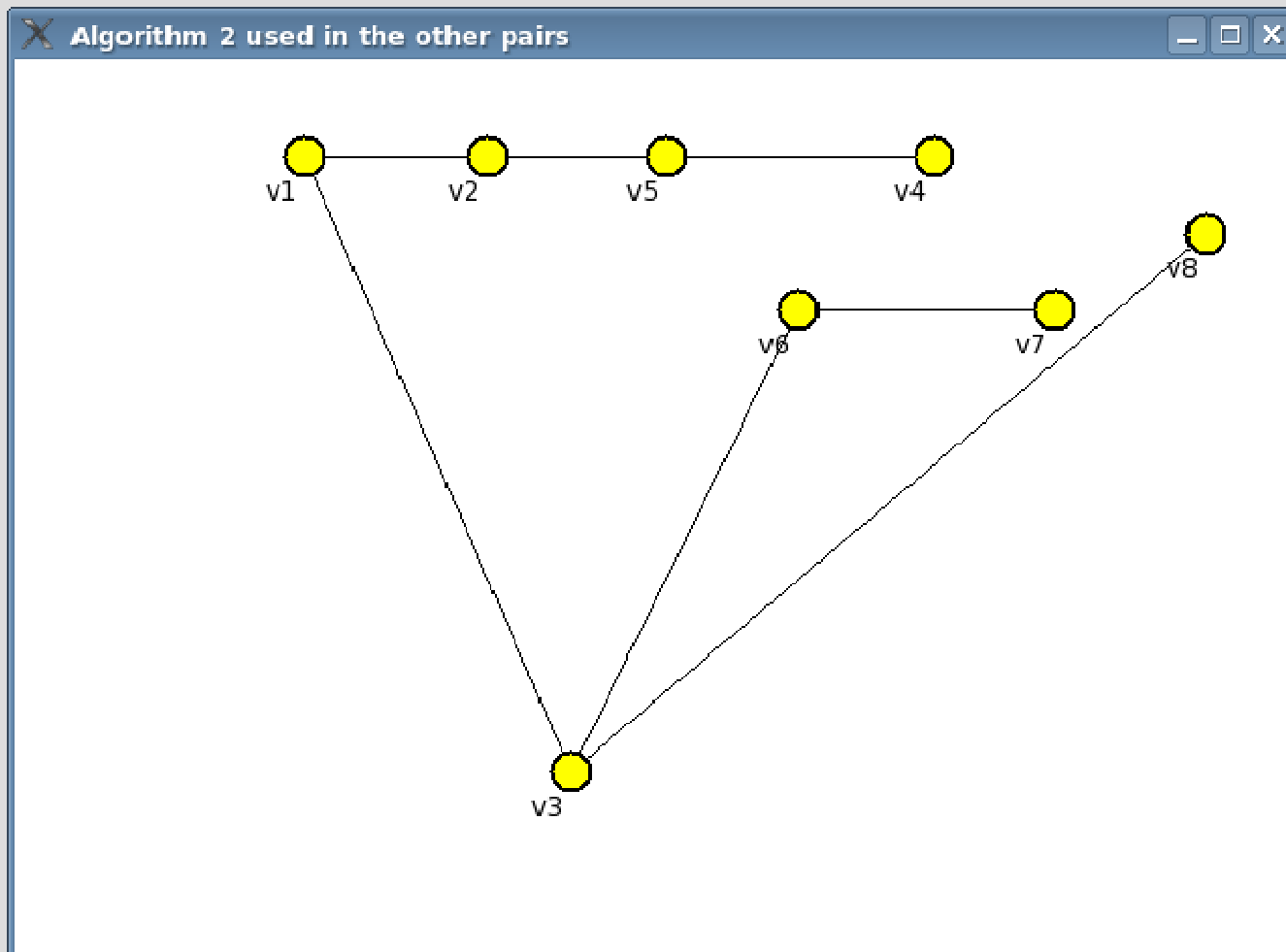
Computing the position of vertices in the last column



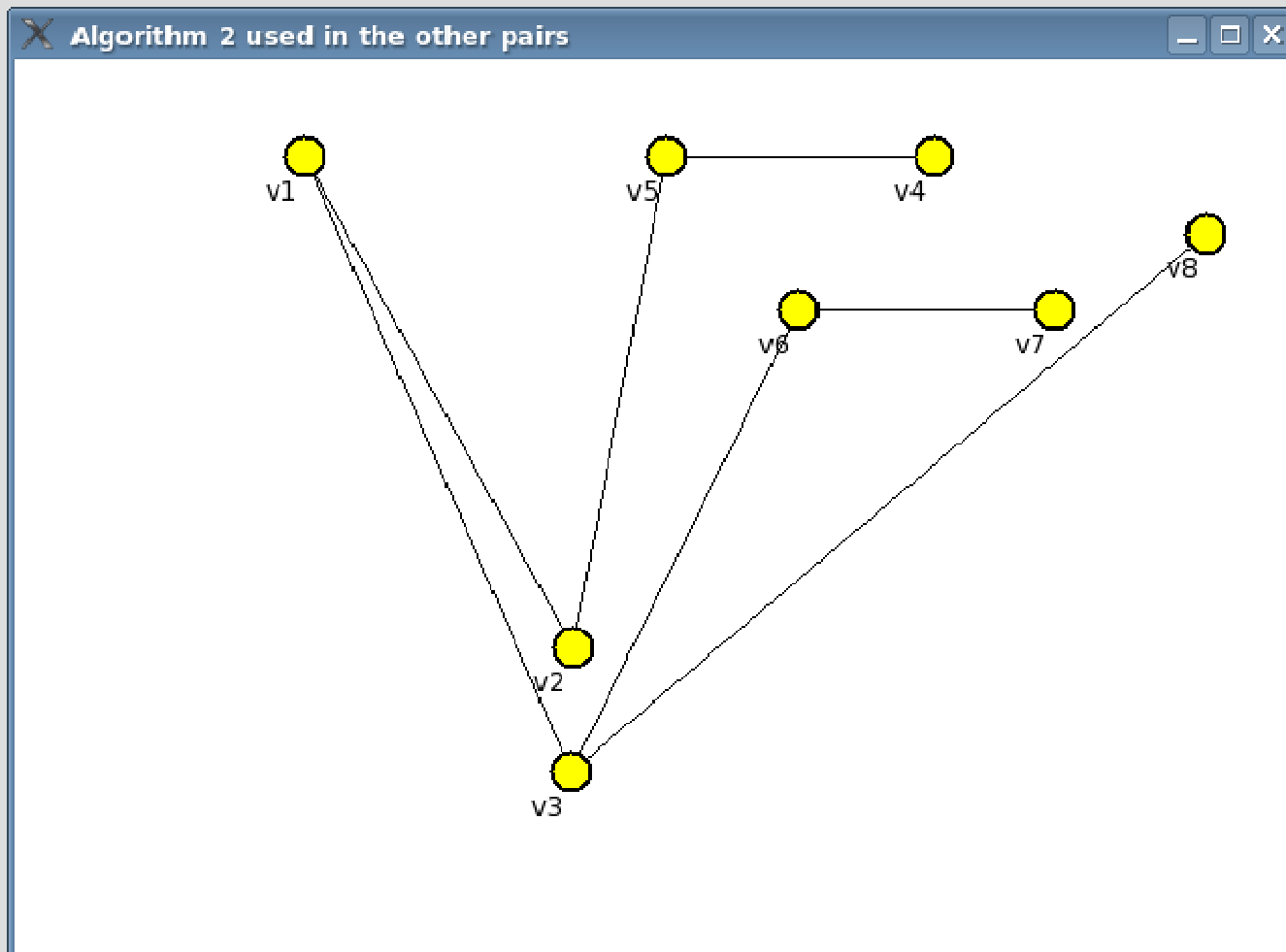
Ordering in Algorithm 2



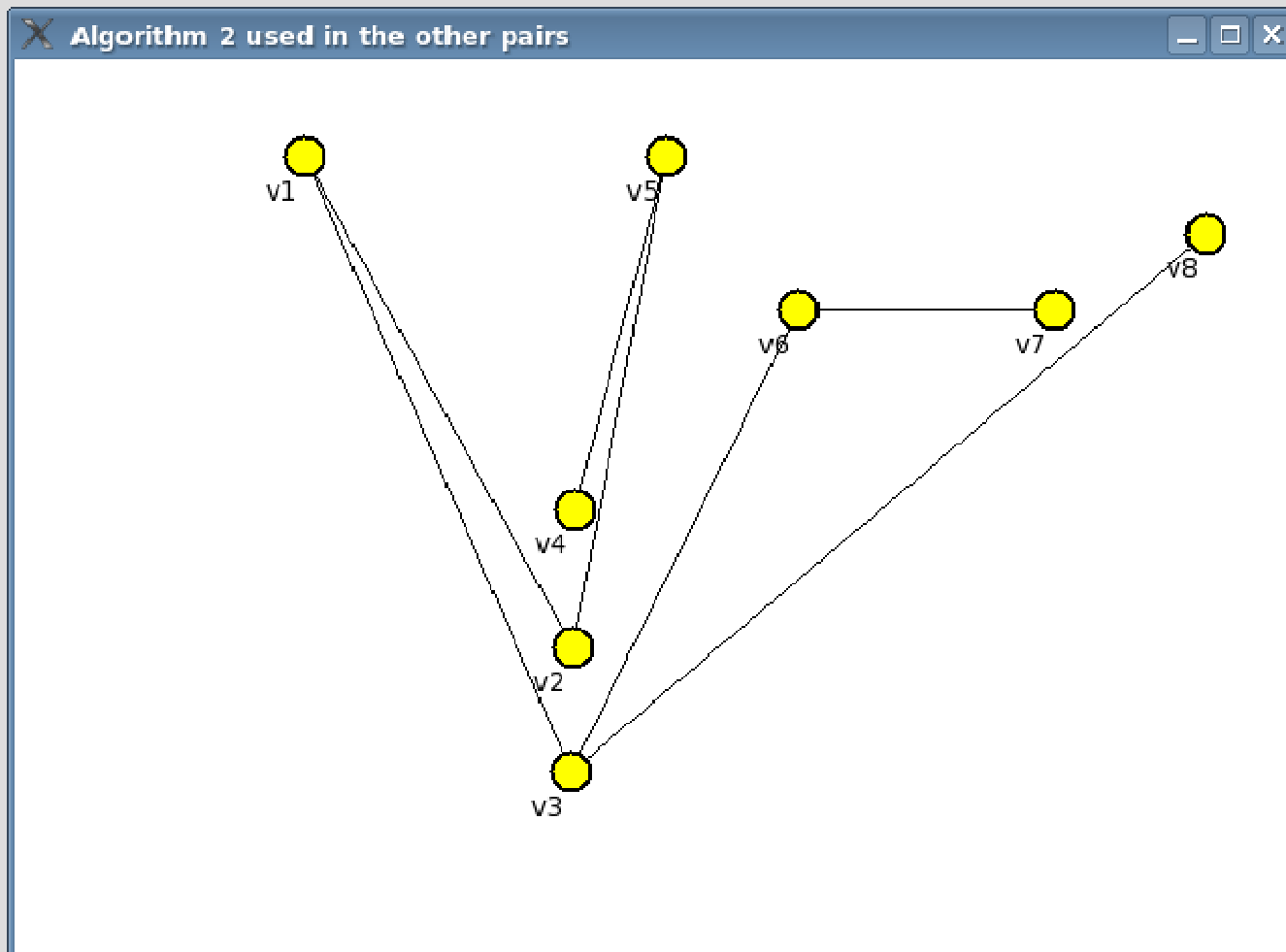
Computing the vertices in 1-st column



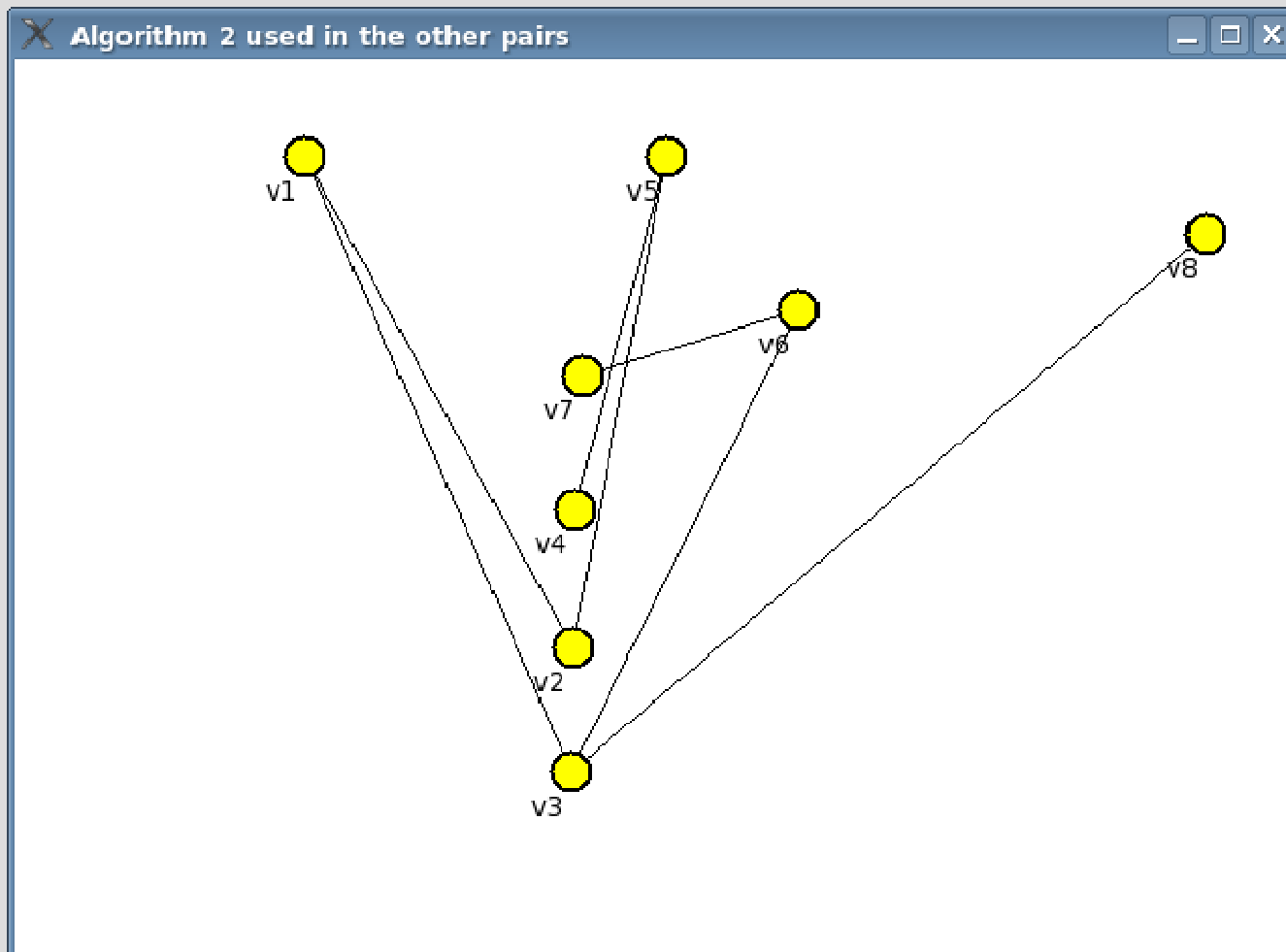
Computing the vertices in 1-st column



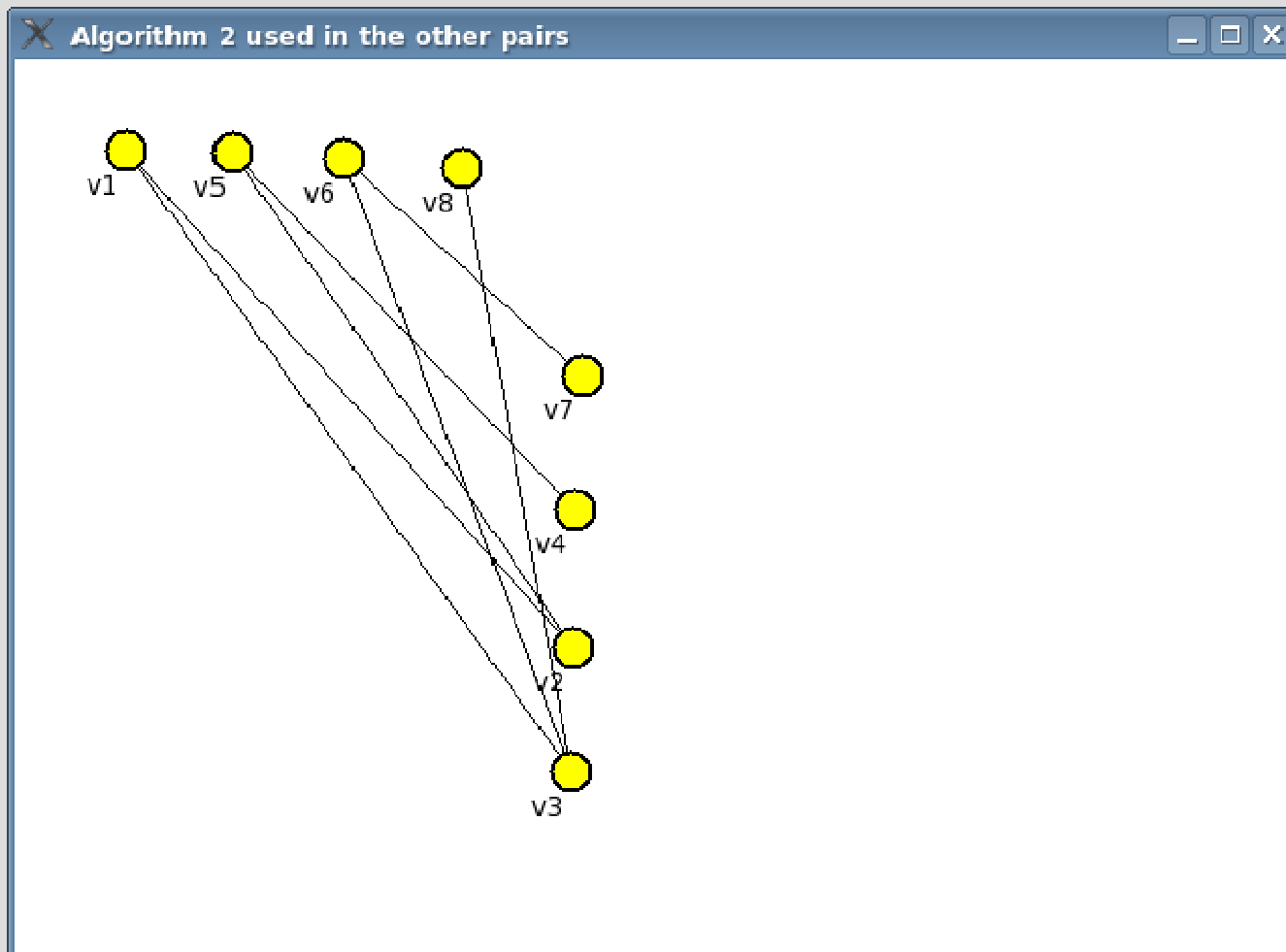
Computing the vertices in 1-st column



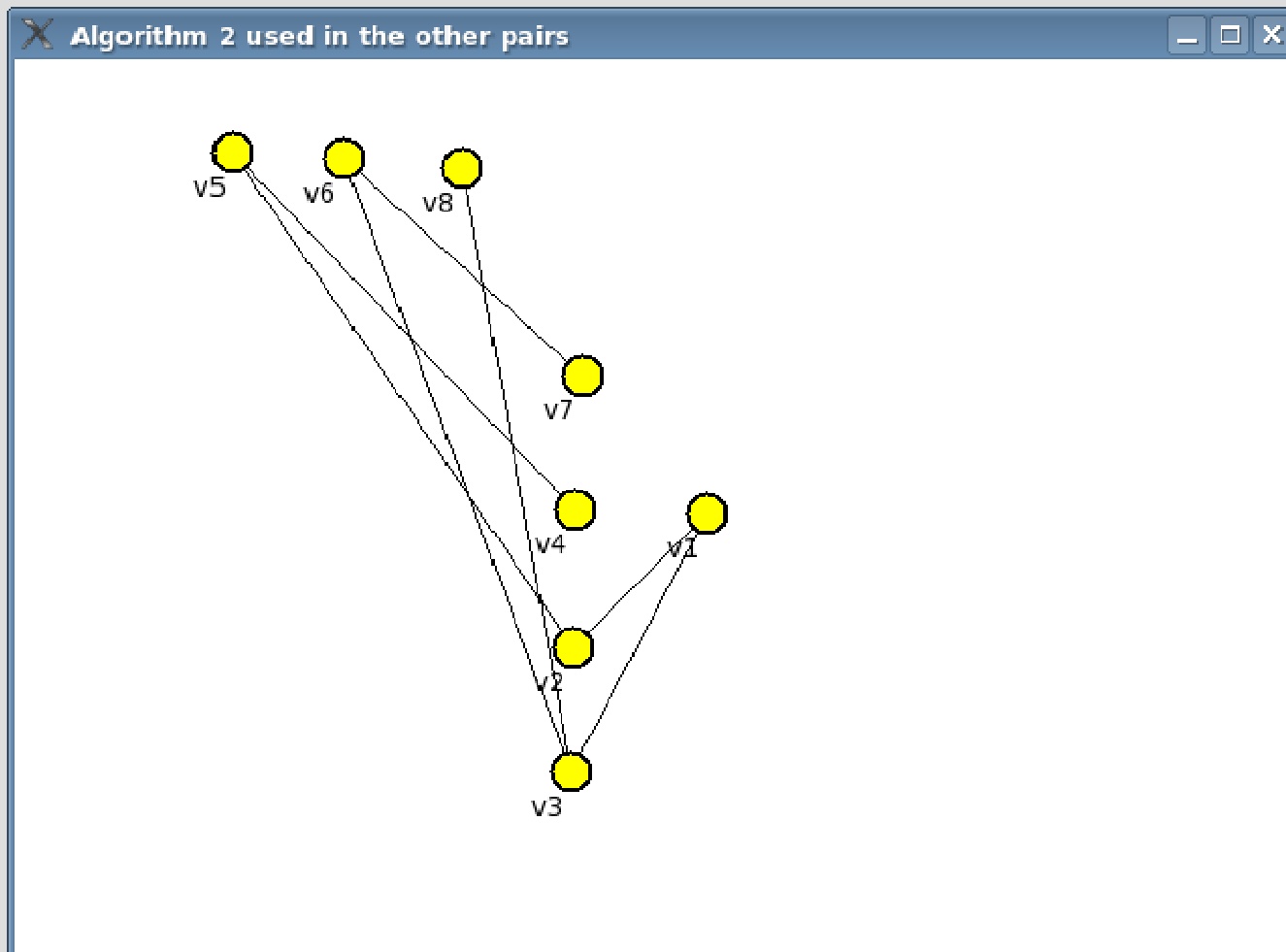
Computing the vertices in 1-st column



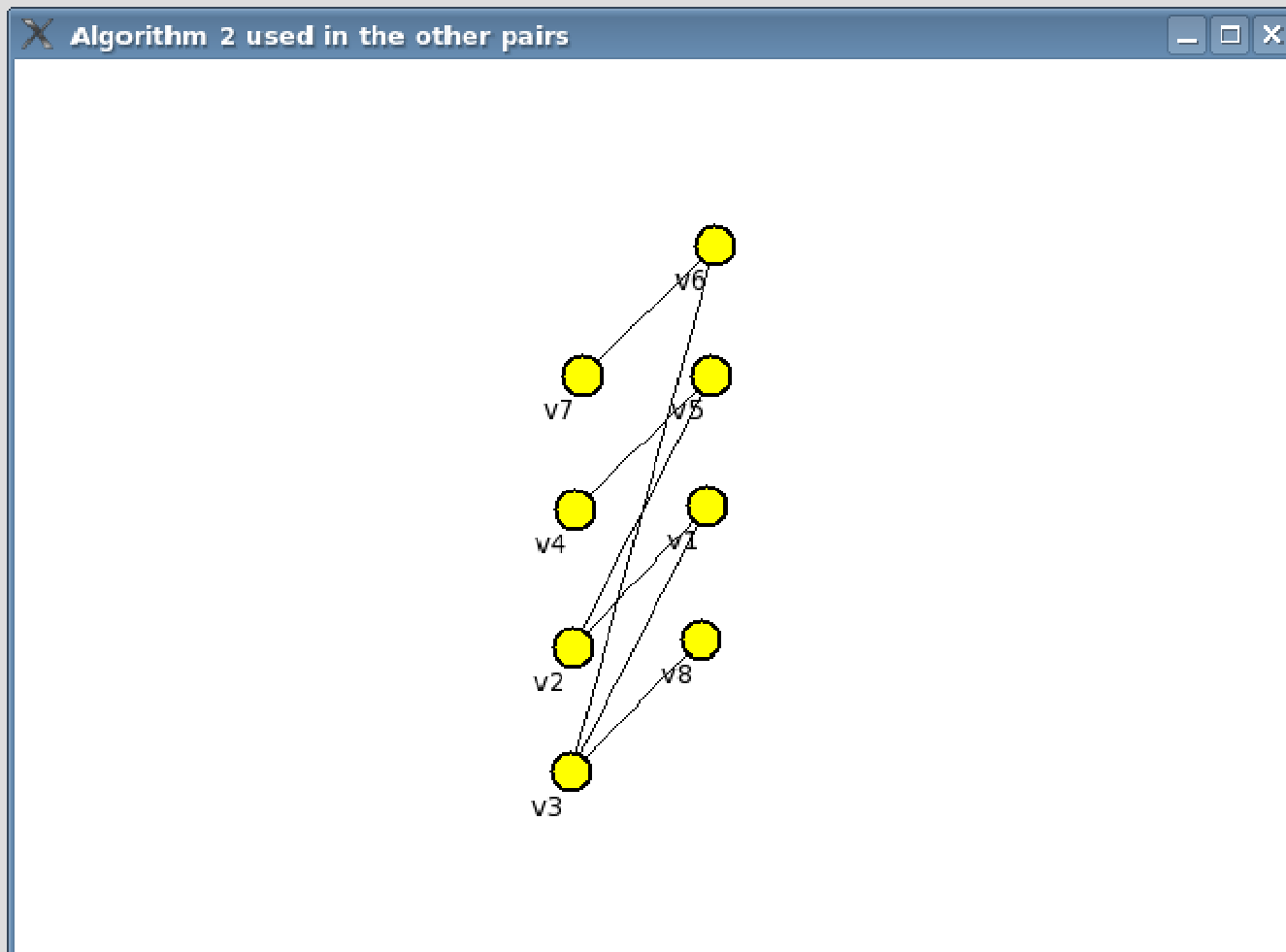
Reordering



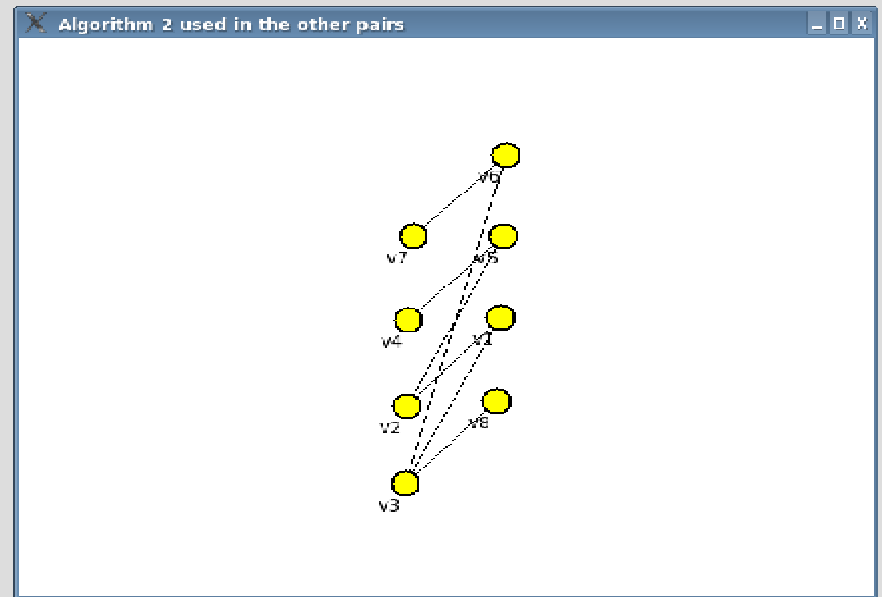
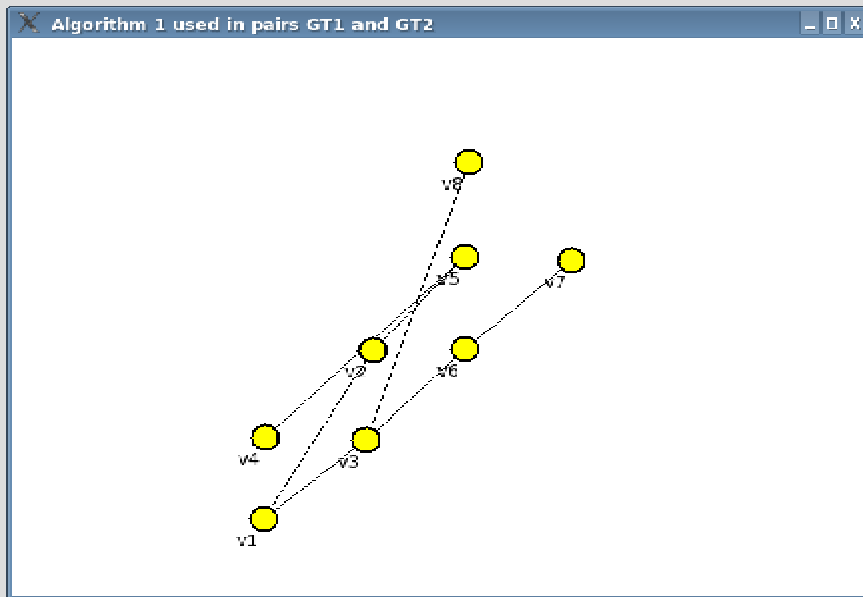
Computing the vertices in 2-nd column



Computing the vertices in 2-nd column



Comparing grids from Algorithm 1 and 2



Binary Coding

Coding of the vertices

$$V_1 \rightarrow 00\dots01(\mathbf{b}) = 2^{1-1}(\mathbf{d})$$

$$V_2 \rightarrow 00\dots10(\mathbf{b}) = 2^{2-1}(\mathbf{d})$$

$$V_3 \rightarrow 0\dots100(\mathbf{b}) = 2^{3-1}(\mathbf{d})$$

.....

$$V_n \rightarrow 1000\dots(\mathbf{b}) = 2^{n-1}(\mathbf{d})$$

Operations with lists in binary form

Coding of the lists :

list = [1,3,6,7] -> b_list = 1100101(b)

Adding item in list in binary form:

b_list = b_list OR b_item

b_list ***1100101(b)***

OR

b_item ***0010000(b)***

b_list ***1110101(b)***

Operations with lists in binary form

Removing item from list in binary form:

$b_list = b_list \text{ AND NOT } b_item$

b_item ***0010000(b)***

b_list ***1110101(b)***

AND

NOT b_item ***1101111(b)***

b_list ***1100101(b)***

Operations with lists in binary form

Union of lists in binary form without repeating of the items:

$$b_list1 = b_list1 \text{ OR } b_list2$$

b_list1 ***1100101(b)***

OR

b_list2 ***1001110(b)***

b_list1 ***1101111(b)***

Operations with lists in binary form

Intersection of lists in binary form:

$$b_list3 = b_list1 \text{ AND } b_list2$$

b_list1 ***1100101(b)***

AND

b_list2 ***1001110(b)***

b_list3 ***1000100(b)***

Operations with lists in binary form

Difference of lists in binary form:

$$b_list3 = b_list1 \text{ AND NOT } b_list2$$

b_list2 ***1001110(b)***

b_list1 ***1100101(b)***

AND

NOT b_list2 ***0110001(b)***

b_list3 ***0100001(b)***

Lookup Table

000000000000000001 -> 1

000000000000000010 -> 1

000000000000000011 -> 2

000000000000000100 -> 1

.....

111111111111111110 -> 15

111111111111111111 -> 16

Results

Random graphs

Name	Cliquer time	ACF time + GT1a file	ACF time + GT1b file	ACF time + GT2a file	ACF time + GT2b file	ACF time + GT3a file	ACF time + GT3b file	ACF time + GT4a file	ACF time + GT4b file
g1000-30-50.b	8.42	61.48	61.47	129.18	13.91	154.8	155.69	36.35	6.94
g1000-35-55.b	13.85	101.2	100.94	1800++	137.44	1800++	1800++	367.97	34.49
g1000-40-55.b	14.31	55.39	55.46	1800++	269.62	909.19	906	93.29	33.72
g1000-45-55.b	3.52	47.83	47.84	1598.9	294.7	539.89	547.92	104	14.75
g1000-50-55.b	5.79	33	32.9	1800++	73.05	665.89	23.48	50.67	5.72
g1000-35-60.b	129.01	1800++	1800++	1800++	883.67	1800++	547.92	1800++	242.74
g1000-40-60.b	138.57	885.76	884.96	1800++	886.12	1800++	1800++	1800++	144.18
g1000-45-60.b	15.88	433.01	431.62	1800++	1800++	1273.12	456.85	1326	49.08
g1000-50-60.b	12.85	384.16	383.72	1800++	1414	1800++	642.35	327.67	23.05
G1000-40-65.b	1784	1800++	1800++	1800++	1800++	1800++	1800++	1800++	1800++
g1000-45-65.b	1800++	1800++	1800++	1800++	1800++	1800++	1800++	1800++	1189+44.18s
g1000-50-65.b	189.06	1800++	1800++	1800++	1800++	1800++	1800++	1800++	368.42
g1000-75-65.b	32.77	68.36	68.08	1800++	1800++	1800++	1150.43	279.22	49.06

Random graphs

Name	Cliquer time	ACF time + GT1a file	ACF time + GT1b file	ACF time + GT2a file	ACF time + GT2b file	ACF time + GT3a file	ACF time + GT3b file	ACF time + GT4a file	ACF time + GT4b file
g1000-75-70.b	984.1	1241	1247	1800++	1800++	1800++	1800++	1800++	264.16+49
g1000-100-70.b	44.08	140.95	140.62	1800++	1800++	1800++	1800++	1734	32.91+48.98
g1000-120-70.b	6.54	28.45	28.58	1800++	1800++	1800++	436.27	115.29	5.48+49.36
g1000-140-70.b	1.63	9.48	9.59	1800++	1800++	133.07	29.97	14.05	0.63+49.03
g1000-100-75.b	648.87	1800++	1800++	1800++	1800++	1800++	1800++	1800++	1228.12+54.21
g1000-120-75.b	1138	576.06	575.06	1800++	1800++	1800++	1800++	1800++	164.53+54.38
g1000-140-75.b	49.35	120.86	121.23	1800++	1800++	1800++	1800++	1575	22.65+58.21
g1000-150-75.b	61.67	74.44	74.55	1800++	1800++	1800++	518.8	94.23	12.54+53.95
g1000-160-75.b	69.3	34.02	33.97	1800++	1800++	1800++	154.12	75.52	2.01+54.92
g1000-170-75.b	1.22	25.4	25.35	1800++	1800++	199.64	24.53	13.2	0.48+55.8
g1000-170-80.b	200.29	805.85	807.09	1800++	1800++	1800++	1800++	1800++	23.06+61.91
g1000-180-80.b	187.55	393.57	393.64	1800++	1800++	1800++	1143.68	462.8	4.57+60.74
g1000-200-80.b	5.99	109.37	109.62	1800++	1800++	557.84	28.82	8.91	0.49+61
g1000-250-85.b	4.35	737.24	736.87	1800++	1800++	123.14	4.88	0.75	0.26+70.29
g1000-300-90.b	1.18	1800++	1800++	1800++	1800++	1800++	17.42	1.87	0.33+81.83
G1000-370-94.b	1.96	1800++	1800++	1800++	1800++	1800++	1.98	0.3	0.27+95.25s
G1000-365-94.b	2.88	1800++	1800++	1800++	1800++	415.16	4.36	0.75	0.29+95.16s
G1000-360-94.b	894.13	1800++	1800++	1800++	1800++	1800++	71.39	1.98	0.35+95.07
g1000-400-95.b	1.96	1800++	1800++	1800++	1800++	1.98	0.38	0.26	0.23+101.29
G1000-370-95.b	1800++	1800++	1800++	1800++	1800++	1800++	1800++	1800++	22.63+101.02
G1000-375-95.b	204.44	1800++	1800++	1800++	1800++	1800++	766.67	571.85	2.43+99.31
g1000-380-95.b	556.51	1800++	1800++	1800++	1800++	1800++	64.74	102.61	1.72+100.86
G1000-385-95.b	19.39	1800++	1800++	1800++	1800++	1800++	14.7	2.2	0.38+100.97
G1000-390-95.b	7.44	1800++	1800++	1800++	1800++	877.68	1.86	0.31	0.29+100.21
G1000-395-95.b	2.79	1800++	1800++	1800++	1800++	49.45	1.09	0.3	0.26+110.97
G1000-400-96.b	1800++	1800++	1800++	1800++	1800++	1800++	152.06	76.76	1.42+105.34
G1000-405-96.b	1070	1800++	1800++	1800++	1800++	1800++	121.47	24.57	0.64+127.04
G1000-410-96.b	1412.79	1800++	1800++	1800++	1800++	1800++	83.57	2.66	0.4+105.79

Random graphs

Name	Cliquer time	ACF time + GT4b file
g1000-75-70.b	984.1	264.16
g1000-100-70.b	44.08	32.91
g1000-120-70.b	6.54	5.48
g1000-140-70.b	1.63	0.63
g1000-100-75.b	648.87	1228.12
g1000-120-75.b	1138	164.53
g1000-140-75.b	49.35	22.65
g1000-150-75.b	61.67	12.54
g1000-160-75.b	69.3	2.01
g1000-170-75.b	1.22	0.48
g1000-170-80.b	200.29	23.06
g1000-180-80.b	187.55	4.57
g1000-200-80.b	5.99	0.49
g1000-250-85.b	4.35	0.26
g1000-300-90.b	1.18	0.33
G1000-370-94.b	1.96	0.27
G1000-365-94.b	2.88	0.29
G1000-360-94.b	894.13	0.35
g1000-400-95.b	1.96	0.23
G1000-370-95.b	1800++	22.63
G1000-375-95.b	204.44	2.43
g1000-380-95.b	556.51	1.72
G1000-385-95.b	19.39	0.38
G1000-390-95.b	7.44	0.29
G1000-395-95.b	2.79	0.26
G1000-400-96.b	1800++	1.42
G1000-405-96.b	1070	0.64
G1000-410-96.b	1412.79	0.4

DIMACS graphs

Name of File	V	E	D(graph)	Max clique	ACF time + GT1a file	ACF time + GT1b file	ACF time + GT2a file	ACF time + GT2b file	ACF time + GT3a file	ACF time + GT3b file	ACF time + GT4a file
brock400_2.clq	400	59786	0,75	29	13486	13393	9719	30284	17259	13451	15229+2.25s
brock400_3.clq	400	59681	0,75	31	7874	7885	5916	11715	1315	14313	625.34+2.39s
brock400_4.clq	400	59765	0,75	33	3801	3775	3519	8124	515	1617	191.69+2.31s
p_hat500-2.clq	500	62946	0,50	36	194.32	194.28	192.15	191.87	132,59	23.63+1.03s	187,46
san400_0.7_1.clq	400	55860	0,70	40	10++h	10++h	10++h	10++ h	10++h	10++h	3490
san400_0.7_2.clq	400	55860	0,70	30	10++h	10++h	10++h	10++ h	10++h	783	99,3
Sanr200_0.9.clq	200	17963	0,90	42	10++h	10++h	10++h	10++ h	10++h	30573	10++ h
MANN_a27.clq	378	70551	0,99	126	138,81+1.14s	138,42+1.18s	10++ h	10++ h	10++h	138,75	10++ h
sanr400_0.7.clq	400	55869	0,70	21	10++ h	10++h	10++h	10++ h	6948	4768	5036
Brock400_1.clq	400	59723	0,75	27	10++ h	10++h	10++h	10++ h	10++h	10++h	31602
Brock800_4.clq	800	207643	0,65	26	10++ h	10++h	10++ h	10++ h	10++h	10++h	25525